Section 3-3, Mathematics 104

Percent Problems

Whenever dealing with percents, remember that a percent is just a fraction

$$5\% = \frac{5}{100} = \frac{1}{20}$$
$$10\% = \frac{10}{100} = \frac{1}{10}$$
$$50\% = \frac{50}{100} = \frac{1}{2}$$

If you have a fraction and want to turn it into a percent, change the denominator to 100

$$\frac{1}{7} = \frac{x}{100}$$

To solve this, multiply both sides by 100

$$x = \frac{100}{7} \approx 14.28\%$$

Example:

What number is 30% of 70

Translate

What number is $\frac{30}{100} = \frac{3}{10}$ of 70.

$$x = \frac{3}{10} \cdot 70 = 21$$

Example:

Your boss offers you a 10% increase in your salary.
Your old salary is \$45,000.
What will your new salary be.

Increase = 10% x \$45,000= $\frac{10}{100} \cdot 45,000 = 4,500$

45,000 + 4,500 = 49,500.

Example:

14 is 25% of what number.

$$x \cdot 25\% = 14$$
$$x \cdot \frac{25}{100} = 14$$
$$x = 14 \cdot \frac{100}{25} = 56$$

Example:

135 is what percent of 27.

Here is the ratio to set up

$$\frac{135}{27} = \frac{P}{100}$$

Solving this by multiplying both sides by 100 gives

$$P = 100 \cdot \frac{135}{27} = 500\%$$

Mark up and Mark down

Example:

The cost of a product is \$45. The store marks it up 55%. What is the price?

Mark up $\$45 \times 55\% = 45 \times \frac{55}{100} = 24.75$

Final cost: $\$45 + \$24.27 = \$69.27$

Example:

A \$3.49 bottle of suntan oil is advertised at 40% off. What is the sale price

$$\$3.49 - (\$3.49 \cdot 40\%) =$$
$$\$3.49 - \left(\$3.49 \cdot \frac{40}{100}\right) = \$3.49 - \$1.40 = \$2.09$$

Example:

The original price of a lawn mower was \$199.95.
On sale the lawn mower costs \$139.95.
What is the discount rate.

Discount amount: $\$199.95 - \$139.95 = \$60$

Discount rate: $\dfrac{\$60}{\$199.95} \approx .30$

Discount Percent: $.30 \times 100 = 30\%$

Section 3-4, Mathematics 104

Ratios

A ratio is a comparison of one number to another by division.

We've just finished talking about percents.

A percent is just a ratio of a number and 100.

So like a percent, a ratio can be expressed as just a number.

Sometimes a ratio is written as a single number .eg 5 or .2 or 3/4.
In this case you can think of it as 5 to 1, .2 to 1 or 3/4 to 1.

Occasionally you may see a ratio is written this way 6:7.

Examples:

The ratio of 7 to 5 is $\frac{7}{5}$

The ratio of 10 to 2 is $\frac{10}{2} = \frac{5}{1} = 5$

A ratio is always a pure number without units.

If your example you compare to containers of liquid

$\frac{5\ gallons}{7\ gallons}$ since both numbers have the same units, you can drop them as if they were a number multiplying the fraction.

Using units this way is called **unit analysis**.

Unit Prices

When you are in the grocery store, you will often see unit prices for items.
The unit prices is just the price for one unit of product even though you often can't by just one unit.

The reason unit pricing is mandated by law in many places is that it helps people compare the value of what you are buying.

Example:

You see to containers of oatmeal in the store.

1 - 75 ounces for $2.39

2 - 200 ounces for $5.47

Which is the better value.

The unit price for 1 is $\frac{\$2.39}{75} = .0319$ dollars per ounce

The unit price for 2 is $\frac{\$5.47}{200} = .0274$ dollars per ounce

So clearly 2 is the better buy. Usually the more you buy, the better the value, however this is not always the case.

Solving Proportions

Often we have a reason to compare to ratios, eg.

$$\frac{x}{5} = \frac{3}{7}$$

A digression about cross multiplying

You may already know about cross multiplying.   In the above example we can cross multiply and get

$$7x = 5 \cdot 3$$
$$x = 15/7$$

It is good to know why this works, so here goes.

$$\frac{a}{b} = \frac{c}{d}$$

We multiply both sides of this equation by *bd*, which we can do since neither *b* nor *d* can be zero.

$$bd\frac{a}{b} = bd\frac{c}{d}$$

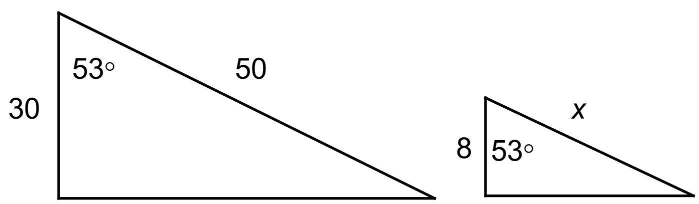
On the left the *b*'s cancel and on the right the *d*'s cancel leaving

$$ad = bc$$

Similar Triangles

In geometry we have the idea of **similar** triangles.
If you can show that two triangles have two equal angles, then their third angle must also be equal and the triangles are similar.
Similar triangles have the property that the ratios of corresponding sides are equal.

Example:



These two right triangles each have a 53 degree angle.
Since two of the angles are equal, the triangles are similar.
The vertical leg of the larger triangle is length 30 and its hypotenuse is 50.
The smaller triangle's vertical leg is 8.
What is the length of the hypotenuse of the smaller triangle?

The ratio 30:8 = 50:x

so $\frac{30}{8} = \frac{50}{x}$

Cross multiplying we get

$$50 \cdot 8 = 30x$$
$$400 = 30x$$
$$x = \frac{400}{30} = 13.33$$

Section 3-5, Mathematics 104

A Mixture Problem

You have \$10,000 to spend.

You invest part of the money at 4.5% and the rest at 5.5% interest.
After one year you have \$508.75 in interest.
How much did you put in each investment?

This seems like a hard problem because there are two different amounts you invest.
If you look at the problem properly it is easy.

$P1$ - the amount you invest at 4.5%

$P2$ - the amount you invest at 5.5%.
But notice that $P2$=10,000-$P1$

So the equation is

$$P1(4.5\%)+(10,000-P1)(5.5\%)=508.75$$

$$\frac{4.5P1}{100}+10,000\cdot\frac{5.5}{100}-\frac{5.5P1}{100}=508.75$$

Multiplying through by 100 we get

$$4.5P1+55,000-5.5P1=50875$$

Subtracting 50875 from both sides and adding $P1$ to both sides we get

$$4134=1.5P1$$

Now dividing both sides by 1.5 we get \$4125.

So the amounts are $\$4125(4.5\%)+\$5875(5.5\%)$

Checking we find this adds up to exactly \$508.75